Fall 2013 **Math 601** Name: ____________________

Quiz 6

Question 1. (5 pts)

Verify that the rotation matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is an orthogonal matrix.

Solution: By definition, we only need to check that

$$AA^T = A^T A = I_2$$

This is a straightforward calculation. I leave it for you to verify the details.

Question 2. (5 pts)

Let $u_1 = (1, 0, 1)$, $u_2 = (-1, 0, 1)$ and $u_3 = (0, 1, 0)$. We know that $S = \{u_1, u_2, u_3\}$ is an orthogonal basis of $\mathbb{R}^3$. Now suppose $v = (0, 3, 4)$. Find the coordinates of v with respect to the basis S.

Solution: Since u_1, u_2 and u_3 are orthogonal, we can use the formula

$$v = \frac{\langle v, u_1 \rangle}{\langle u_1, u_1 \rangle} u_1 + \frac{\langle v, u_2 \rangle}{\langle u_2, u_2 \rangle} u_2 + \frac{\langle v, u_3 \rangle}{\langle u_3, u_3 \rangle} u_3 = 2u_1 + 2u_2 + 3u_3$$

It follows that

$$[v]_S = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

Question 3. (10 pts)

Let U be the subspace of $\mathbb{R}^4$ spanned by $v_1 = (1, 1, 1, 1), v_2 = (1, 1, 2, 4)$ and $v_3 = (1, 2, -4, -3)$. Use the Gram-Schmidt process to find an *orthonormal* basis of U.

Solution: This is Exercise 7.21 on Page 249 of the textbook.